Contents

| Pref | face | | 1 | v |
|------|------------------------------------|---|--|--|
| Pref | face t | o the E | English edition | vi |
| Ack | nowl | edgeme | nts | vii |
| Inti | rodu | ction | | 1 |
| 1 | Interpolation and Hörmander spaces | | | |
| | 1.1 | Interp 1.1.1 1.1.2 1.1.3 1.1.4 1.1.5 1.1.6 1.1.7 1.1.8 1.1.9 | olation with function parameter Definition of interpolation Embeddings of spaces Reiteration property Interpolation of dual spaces Interpolation of orthogonal sums of spaces Interpolation of subspaces and factor spaces Interpolation of Fredholm operators Estimate of the operator norm in interpolation spaces Criterion for a function to be an interpolation parameter | 9 9 11 13 15 18 20 21 23 25 |
| | 1.2 | Regula 1.2.1 1.2.2 1.2.3 | arly varying functions and their generalizationRegularly varying functionsQuasiregularly varying functionsAuxiliary results | 29 29 31 36 |
| | 1.3 | Hörma 1.3.1 1.3.2 1.3.3 1.3.4 | ander spaces and the refined Sobolev scale Preliminary information and notation Hörmander spaces Refined Sobolev scale Properties of the refined scale | 38 38 40 42 44 |
| | 1.4 | Unifor 1.4.1 1.4.2 1.4.3 | cmly elliptic operators on the refined scalePseudodifferential operatorsA priori estimate of the solutionsSmoothness of the solutions | 47 47 50 51 |
| | 1.5 | Rema | rks and comments | 55 |
| 2 | Hö | rmand | er spaces on closed manifolds and their applications | 59 |
| | 2.1 | Hörma 2.1.1 | ander spaces on closed manifolds | 59 59 |



| | | 2.1.2 2.1.3 2.1.4 | Interpolation properties 6 Equivalent norms 6 Embedding theorem 7 | 51 58 77 |
|---|---------------------------------|---|---|--|
| | 2.2 | Ellipti 2.2.1 | c operators on closed manifolds | 78 79 |
| | | 2.2.2 2.2.3 2.2.4 | Elliptic operators on the refined scale | 81 84 86 |
| | 2.3 | Conve 2.3.1 | rgence of spectral expansions | 93 |
| | | 2.3.2 2.3.3 | Convergence almost everywhere for spectral expansions . So Convergence of spectral expansions in the metric of the space C^k | 95 95 97 |
| | 2.4 | RO-va 2.4.1 2.4.2 2.4.3 | arying functions and Hörmander spaces 9 RO-varying functions in the sense of Avakumović 9 Interpolation spaces for a pair of Sobolev spaces 10 Applications to elliptic operators 10 | 98 98 00 07 |
| | 2.5 | Rema | rks and comments | 08 |
| | | | | |
| 3 | Sen | nihomo | ogeneous elliptic boundary-value problems 11 | 11 |
| 3 | Sen 3.1 | nihomo Regula 3.1.1 3.1.2 | ogeneous elliptic boundary-value problems 11 ar elliptic boundary-value problems 11 Definition of the problem 11 Formally adjoint problem 11 | 11 11 11 13 |
| 3 | Sen 3.1 3.2 | hihomo Regula 3.1.1 3.1.2 Hörma 3.2.1 3.2.2 3.2.3 | ogeneous elliptic boundary-value problems11ar elliptic boundary-value problems11Definition of the problem11Formally adjoint problem11ander spaces for Euclidean domains11Spaces for open domains11Spaces for closed domains12Rigging of $L_2(\Omega)$ with Hörmander spaces12 | $ \begin{array}{r} 11 \\ 11 \\ 13 \\ 14 \\ 15 \\ 20 \\ 23 \\ \end{array} $ |
| 3 | Sen 3.1 3.2 3.3 | nihomo Regula 3.1.1 3.1.2 Hörma 3.2.1 3.2.2 3.2.3 Bound 3.3.1 3.3.2 3.3.3 3.3.4 3.3.5 | ogeneous elliptic boundary-value problems11ar elliptic boundary-value problems11Definition of the problem11Formally adjoint problem11ander spaces for Euclidean domains11spaces for open domains12Spaces for closed domains12Rigging of $L_2(\Omega)$ with Hörmander spaces12dary-value problems for homogeneous elliptic equations12Main result: boundedness and Fredholm property of the operator12A theorem on interpolation of subspaces12Proof of the main result13Properties of solutions to the homogeneous elliptic equation13equation13 | 11 11 11 13 14 15 20 23 26 26 27 31 34 39 |
| 3 | Sen 3.1 3.2 3.3 3.3 | nihomo Regula 3.1.1 3.1.2 Hörma 3.2.1 3.2.2 3.2.3 Bound 3.3.1 3.3.2 3.3.3 3.3.4 3.3.5 Ellipti | ogeneous elliptic boundary-value problems11ar elliptic boundary-value problems11Definition of the problem11Formally adjoint problem11ander spaces for Euclidean domains11spaces for open domains11Spaces for closed domains12Rigging of $L_2(\Omega)$ with Hörmander spaces12lary-value problems for homogeneous elliptic equations12Main result: boundedness and Fredholm property of the operator12Proof of the main result13Properties of solutions to the homogeneous elliptic equation13equation14equation15and result homogeneous boundary conditions14 | 111 111 113 114 115 200 233 226 226 227 311 34 39 422 42 |

| | | 3.4.3 | Proofs of theorems on isomorphisms and the Fredholm property | 152 |
|---|------|--|--|---|
| | | 3.4.4 | Local increase in smoothness of solutions up to the boundary | 156 |
| | 3.5 | Some 3 3.5.1 3.5.2 | properties of Hörmander spaces \dots Space $H_0^{s,\varphi}(\Omega)$ and its properties \dots Equivalent description of $H^{s,\varphi}(\Omega)$ | 158 158 160 |
| | 3.6 | Remar | ks and comments | 162 |
| 4 | Inh | omoge | neous elliptic boundary-value problems | 165 |
| | 4.1 | Ellipti 4.1.1 4.1.2 4.1.3 4.1.4 4.1.5 | c boundary-value problems in the positive one-sided scale Theorems on Fredholm property and isomorphisms Smoothness of the solutions up to the boundary Nonregular elliptic boundary-value problems Parameter-elliptic boundary-value problems Formally mixed elliptic boundary-value problem | 165 165 169 173 175 186 |
| | 4.2 | Ellipti 4.2.1 4.2.2 4.2.3 4.2.4 4.2.4 | c boundary-value problems in the two-sided scale Preliminary remarks The refined scale modified in the sense of Roitberg Roitberg-type theorems on solvability. The complete collection of isomorphisms Smoothness of generalized solutions up to the boundary . Interpolation in the modified refined scale | 188 189 199 204 207 |
| | 4.3 | Some 4.3.1 4.3.2 | properties of the modified refined scale Statement of results Proof of results | 210 210 212 |
| | 4.4 | Genera 4.4.1 4.4.2 4.4.3 4.4.4 | alization of the Lions-Magenes theorems | 226 227 230 236 238 |
| | 4.5 | Hörma 4.5.1 4.5.2 | Ander spaces and individual theorems on solvability Key individual theorem for the refined scale Other individual theorems | 243 243 244 |
| | 4.6 | Remai | rks and Comments | 247 |
| 5 | Elli | ptic sy | /stems | 251 |
| | 5.1 | Unifor 5.1.1 5.1.2 5.1.3 | mly elliptic systems in the refined Sobolev scale Uniformly elliptic systems A priori estimate for the solutions of the system Smoothness of solutions | 251 251 252 253 |

| 5.2 | Elliptic systems on a closed manifold | | 257 |
|--------------|---|--|-----|
| | 5.2.1 | Elliptic Systems | 257 |
| | 5.2.2 | Operator of the elliptic system on the refined scale | 258 |
| | 5.2.3 | Local smoothness of solutions | 262 |
| | 5.2.4 | Parameter-elliptic systems | 264 |
| 5.3 | Elliptic boundary-value problems for systems of equations | | 268 |
| | 5.3.1 | Statement of the problem | 269 |
| | 5.3.2 | Theorem on solvability | 271 |
| 5.4 | Remai | ks and comments | 272 |
| Bibliography | | | 275 |
| Index | | | 291 |

•